Mathematical League of University of Lodz

Series I 24/25

For every exercise you can get max. 10. p. Solutions should be delivered on paper (every task on the separate piece of paper) to the room B207 or electronically on the address: piotr.nowakowski@wmii.uni.lodz.pl. Deadline: 29.11.24.

Exercise 1. *Players $1, 2, \dots, n$ seat around a table and each of them has a single coin. Player 1 passes a coin to player 2, who then gives two coins to player 3. Next, player 3 passes one coin to player 4, who passes two coins to player 5, and so on. So, players in turns pass one coin or two coins to the next player who still has some coins. A player who runs out of coins leaves the game and the table. For what natural numbers n some player finishes with all n coins?*

Exercise 2. *Find all triples of natural numbers (k, n, m) satisfying the following equation*

$$9^k + 12^n = 15^m.$$

Exercise 3. *Let k and n be numbers such that $k < n$. Let Let $S := \{1, 2, \dots, n\}$ and let $A_1, A_2, \dots, A_k$ be nonempty subsets of S (not necessarily pairwise disjoint). We color some elements of S, using two colors red and blue (some may remain uncolored). Prove that there exists a colouring of elements of S such that at least one element of S is colored and each set A_i is either completely uncolored or it contains at least one red and at least one blue element.*